Math 120r
Final Exam Study Guide

The following problems are intended as review of the topics covered in this course and as practice for answering questions in a multiple choice format. These do not represent the actual problems you will see on the final exam.

1. Find the value of c so that the lines $5x + cy = 4$ and $x - 3y = 9$ are perpendicular.

(A) $c = -3$ (B) $c = 15$ (C) $c = -15$ (D) $c = -\frac{5}{3}$ (E) $c = \frac{5}{3}$

① $5x + cy = 4$
$cy = 4 - 5x$
$cy = -5x + 4$
$y = -\frac{5}{c}x + \frac{4}{c}$

② $x - 3y = 9$
$-3y = 9 - x$
$-3y = -x + 9$
$y = \frac{-1}{-3}x + \frac{9}{-3}$
$y = \frac{1}{3}x - 3$

For lines to be perpendicular, the slope of one line is the negative reciprocal of the slope of the other line.

Slope of ② is $\frac{1}{3}$, which means that the slope of ① must be -3.

$-\frac{5}{c} = -3$
$-3c = -5$
$c = \frac{-5}{-3} \rightarrow c = \frac{5}{3}$

2. Find the value of k so that the slope of the line connecting the points $(k, 4)$ and $(-k, 9)$ is 3.

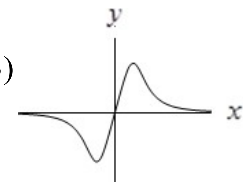
(A) $k = -\frac{15}{2}$ (B) $k = -\frac{6}{5}$ (C) $k = -\frac{5}{6}$ (D) $k = \frac{5}{6}$ (E) $k = \frac{15}{2}$

① $(k, 4)$
② $(-k, 9)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$3 = \frac{9 - 4}{-k - k}$
$3 = \frac{5}{-2k}$
$3(-2k) = 5$
$-6k = 5$
$k = -\frac{5}{6}$

3. Which of the following represent y as a function of x?

(1) $xy + 2 = x$ (2) $x^2 + y^2 = 64$ (3)



(4)

x	y
0	5
2	5
4	5
6	5

(A) (2) only (B) (1), (3), and (4) only (C) (1) and (2) only

(D) (3) and (4) only (E) (1), (2), (3), and (4)

A function has only one output for every input

(1) Function; $\frac{xy+2}{x} = \frac{x}{x} \rightarrow y + \frac{2}{x} = 1 \rightarrow y = -\frac{2}{x} + 1$

(2) Not a function; $x^2+y^2=64 \rightarrow y^2 = 64 - x^2 \rightarrow y = \pm\sqrt{64-x^2} \rightarrow$ y takes 2 different values for each x.

(3) Function; vertical line test

(4) Function; one output for each input

4. Find the x-intercept(s) of the piecewise defined function:

$$f(x) = \begin{cases} -3x+5 & \text{if } x < -5 \\ x^2 - 8 & \text{if } -5 \le x \le -1 \\ \sqrt{x+9} & \text{if } x > -1 \end{cases}$$

(A) $x = -\sqrt{8}$ only (B) $x = -\sqrt{8}$ and $x = \sqrt{8}$ only

(C) $x = \frac{5}{3}$ only (D) $x = \frac{5}{3}$, $x = -9$, $x = -\sqrt{8}$ and $x = \sqrt{8}$ only

(E) There are no x-intercepts.

x-intercepts are where $y = 0$ or $f(x) = 0$.

$0 = -3x + 5$ if $x < -5$
$3x = 5$ if $x < -5$
$x = \frac{5}{3}$ is not $x < -5$
$x = \frac{5}{3}$ is not an x-intercept

$0 = x^2 - 8$ if $-5 \le x \le -1$
$8 = x^2$ if $-5 \le x \le -1$
$\pm\sqrt{8} = x$ if $-5 \le x \le -1$
$+\sqrt{8} = x$ is not $-5 \le x \le -1$
$+\sqrt{8} = x$ is not an x-intercept
$-\sqrt{8} = x \approx -2.83$ is $-5 \le x \le -1$
$-\sqrt{8}$ is an x-intercept

$0 = \sqrt{x+9}$ if $x > -1$
$0^2 = \sqrt{x+9}^2$ if $x > -1$
$0 = x + 9$ if $x > -1$
$-9 = x$ is not $x > -1$
$-9 = x$ is not an x-intercept

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5. Which ONE of the following is the solution to the inequality $y^2(5y+3)(y-6)>0$?

(A) $\left(-\frac{3}{5}, 6\right)$ (B) $\left[-\frac{3}{5}, 6\right]$ (C) $\left(-\infty, -\frac{3}{5}\right) \cup (6, \infty)$

(D) $\left(-\infty, -\frac{3}{5}\right] \cup [6, \infty)$ (E) all real numbers

$y^2 = 0$
$y = 0$

$5y + 3 = 0$
$5y = -3$
$y = -\frac{3}{5}$

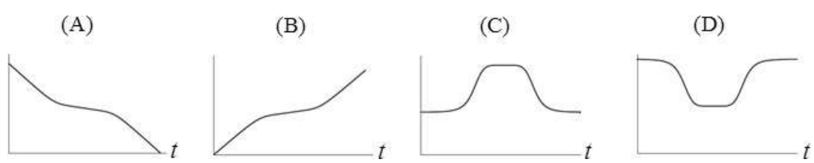
$y - 6 = 0$
$y = 6$

create test intervals to see which ones make the inequality true.

$y < -\frac{3}{5}$ True
$-\frac{3}{5} < y < 0$ False
$0 < y < 4$ False
$y > 6$ True

$y < -\frac{3}{5}$ or $y > 6$
$\left(-\infty, -\frac{3}{5}\right) \cup (6, \infty)$

Use the story and graphs below to answer questions 6 and 7.
As I drove on the freeway this morning, at first traffic was fast as usual, then it moved slowly until we passed a construction area, after which traffic flow went back to normal until I exited the freeway.



6. Which graph above represents my distance from the exit as a function of time on the freeway?
(A) (B) (C) (D)

The distance from the exit will be decreasing as long as they are always moving toward the exit. If they stop for any period of time, the distance will remain constant as a function of time.

7. Which graph above represents my driving speed as a function of time on the freeway?
(A) (B) (C) (D)

The speed starts fast, slows down for some time, and then is fast again.

8. If $f(x)$ is a function with domain $[-8,12]$, find the domain of $\frac{1}{2}f(x-3)$.

(A) $[-5,6]$ (B) $[-7,12]$ (C) $[-5,15]$ (D) $[-11,9]$ (E) $[-13,12]$

The $\frac{1}{2}$ affects the y-values, so it would not affect the domain. The -3 indicates a horizontal shift to the right 3 units. The new domain would be $[-8+3, 12+3]$ which is the same as $[-5, 15]$.

9. Solve for y: $4y^{-2} = y$

(A) $y=\sqrt[3]{4}$ only. (B) $y=0$ only. (C) $y=0$ or $y=\sqrt[3]{4}$ only.

(D) $y=\frac{1}{4}$ only. (E) $y=\sqrt[3]{4}$ or $y=-\sqrt[3]{4}$ only.

$$4y^{-2} = y$$
$$\frac{4}{y^2} = y$$
$$4 = y^3$$
$$\sqrt[3]{4} = y$$

10. Let $g(x) = x+4$. Simplify $[g(x)]^2 - g(x^2)$

(A) $8x+12$ (B) $8x+20$ (C) 12
(D) 0 (E) None of these

$g(x) = x+4$

$[g(x)]^2 - g(x^2) = [x+4]^2 - ((x^2)+4)$
$= [(x+4)(x+4)] - x^2 - 4$
$= [\cancel{x^2} + 8x + 16] - \cancel{x^2} - 4$
$= 8x + 16 - 4$
$= 8x + 12$

11. If $f(x)$ is a one-to-one function, and $f(8) = 11$, then which of the following CANNOT be true?

(A) $f(11) = 8$ (B) $f^{-1}(11) = 8$ (C) $f^{-1}(5) = 3$
(D) $f^{-1}(11) = 5$ (E) $f(-8) = -11$

$f(x)$ is one-to-one
$f(8) = 11 \rightarrow f^{-1}(11) = 8$

$f(8)$ cannot equal anything but 11
$f^{-1}(11)$ cannot equal anything but 8

12. Suppose $g(4) = 30$ means the volume of water in a container is 30 ounces when the depth of the water is 4 inches. What is the meaning of $g^{-1}(50) = 10$?

(A) The volume of the water is 10 ounces when the depth of the water is 50 inches.
(B) The depth of the water is 10 inches when the volume of the water is 50 ounces.
(C) The depth of the water is 0.2 inches when the volume of the water is 50 ounces.
(D) The volume of the water is 5 ounces when the depth of the water is 10 inches.
(E) None of these

input is the depth, output is the volume.
g(input) = output

when taking the inverse, the input is the volume, and the output is the depth. ∴ 50 is the volume in ounces, and 10 is the depth in inches.

13. Consider the functions $f(x)$ and $g(x)$ in the tables below.

x	$f(x)$
5	1
6	4
7	2
8	3

x	$g(x)$
2	-1
3	-4
4	-2
5	-3

Which ONE of the following expresses the correct relationship between $f(x)$ and $g(x)$?

(A) $g(x) = -f(x) + 3$ (B) $g(x) = f(-x+3)$ (C) $g(x) = -f(x+3)$

(D) $g(x) = f(-x) - 3$ (E) $g(x) = -f(x-3)$

$f(5) = 1$ $g(2) = -1$
$f(6) = 4$ $g(3) = -4$
$f(7) = 2$ $g(4) = -2$
$f(8) = 3$ $g(5) = -3$

when the x-value in g(x) is 3 less than the x-value in f(x), the output of g(x) is the negative of f(x).

14. Let $h(x) = -7x + 2$ and $g(x) = x^3$. Find the x-intercept(s) of $h(g(x))$.

(A) $x = -\frac{2}{7}$, $x = \frac{2}{7}$ (B) $x = \frac{2}{7}$ (C) $x = \sqrt[3]{\frac{2}{7}}$

(D) $x = -\sqrt[3]{\frac{2}{7}}$, $x = \sqrt[3]{\frac{2}{7}}$ (E) $x = \sqrt[-3]{\frac{2}{7}}$

x-intercepts are where $h(g(x)) = 0$

$h(g(x)) = -7(g(x)) + 2$

$h(g(x)) = -7(x^3) + 2$

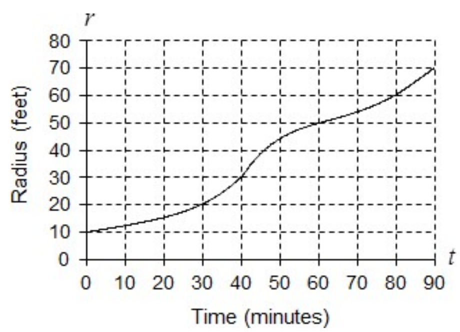
$0 = -7(x^3) + 2$

$0 = -7x^3 + 2$

$7x^3 = 2$ → $x^3 = \frac{2}{7}$

$x = \sqrt[3]{\frac{2}{7}}$

Use the story and graph below to answer questions 15 and 16.
Suppose an oil spill covers a circular area and the radius, r, increases according to the graph shown below where t represents the number of minutes since the spill was first observed.



15. How large is the circular area of the spill 30 minutes after it was first observed?

(A) 900π ft² (B) 1600π ft² (C) 400 ft²
(D) 40π ft² (E) 400π ft²

$r = 20$ feet when $t = 30$ minutes

$A = \pi r^2$

$A = \pi (20ft)^2$

$A = \pi\, 400\, ft^2$

16. If the cost to clean the oil spill is proportional to the square of the diameter of the spill, express the cost, C, as a function of the radius of the spill, r.

(A) $C(r) = k(2r)^{1/2}$ (B) $C(r) = k(2r)^2$ (C) $C(d) = kd^2$
(D) $C(r) = 2kr^2$ (E) $C(r) = 2kr^{1/2}$

$d = 2r$

let k be the constant of proportionality

$C(r) = k(2r)^2$

option C is using the cost as a function of the diameter.

17. If $p(x) = \frac{5}{x}$ and $r(x) = \frac{3}{x^2} - 1$, find $\left(\frac{p}{r}\right)(x)$ for $x \neq 0$.

(A) $\frac{5x}{3 - x^2}$ (B) $\frac{5x - 1}{3}$ (C) $\frac{5}{3 - x^2}$
(D) $\frac{5(x - 1)}{3x}$ (E) $\frac{5x^2}{3 - x^2}$

$P(x) = \frac{5}{x}$

$r(x) = \frac{3}{x^2} - 1$

$\left(\frac{p}{r}\right)(x) = \left(\frac{\frac{5}{x}}{\frac{3}{x^2} - 1}\right)$

$$\frac{\frac{5}{x}}{\frac{3}{x^2} - 1} = \frac{\frac{5}{x}}{\frac{3}{x^2} - \frac{x^2}{x^2}} = \frac{\frac{5}{x}}{\frac{3 - x^2}{x^2}}$$

$$\frac{\frac{5}{x}}{\frac{3 - x^2}{x^2}} = \frac{5}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{3 - x^2} = \frac{5x}{3 - x^2}$$

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18. Find $\frac{g(3+h)-g(3)}{h}$ for $g(t)=4t^2$. Simplify your answer as much as possible.

(A) 1 (B) $24+4h$ (C) $6+h$

(D) $4h$ (E) $\frac{4h^2+24h+36}{h}$

$g(t)=4t^2$
$g(3+h)=4(3+h)^2$
$g(3+h)=4(3+h)(3+h)$
$g(3+h)=4(9+6h+h^2)$
$g(3+h)=36+24h+4h^2$

$g(t)=4t^2$
$g(3)=4(3)^2$
$g(3)=4(9)$
$g(3)=36$

$\frac{g(3+h)-g(3)}{h}$

$=\frac{(36+24h+4h^2)-(36)}{h}$

$=\frac{24h+4h^2}{h}=\frac{\cancel{h}(24+4h)}{\cancel{h}}=24+4h$

19. If the point $(8, -3)$ is on the graph of $f(x)$, find the corresponding point on the graph of the transformation $y = 2f(-x)$.

(A) $(16,6)$ (B) $(4,3)$ (C) $(-16,-3)$ (D) $(-8,-6)$ (E) $(-8,6)$

$f(8)=-3$

① There is no horizontal shift

② There is a vertical stretch

$2\cdot f(8)=2(-3)$
$2f(8)=-6$

③ There is a reflection in the y-axis

$2\cdot f(-(8))=-6$

④ There is no vertical shift

$y=2f(-x)$ gives us $(-8,-6)$

20. If the domain of $g(x)$ is $[-5, 8]$, what is the domain of the function $y = g(x+2) - 1$?

(A) $[-7,6]$ (B) $[-3,9]$ (C) $[-4,6]$ (D) $[-7,7]$ (E) $[-3,10]$

① There is a horizontal shift 2 units to the left

② There is no vertical stretch/compression

③ There is no reflection

④ There is a vertical shift down 1 unit

Since the domain focuses on the x-values, the vertical shift will not affect the domain, but the horizontal shift will. Since the shift is to the left 2 units, we subtract 2 from the beginning and end of our interval.

$[-5-2, 8-2]=[-7,6]$

21. Suppose $x = 6$ is a vertical asymptote of a function $y = h(x)$. Which ONE of the following must be a vertical asymptote of $y = 2h(3x)$?

(A) $x = 6$ (B) $x = 3$ (C) $x = 12$ (D) $x = 18$ (E) $x = 2$

In $y = 2h(3x)$, the 2 indicates a vertical stretch of the graph, which would not affect the vertical asymptote at $x = 6$. The 3 indicates a horizontal compression by a factor of 3. This would make the vertical asymptote at $x = \frac{6}{3}$ or $x = 2$.

22. Which of the following represents the complete factorization of $2(3x+1)^7 - 16x(3x+1)^6$?

(A) $2(3x+1)^6(1+11x)$

(B) $2(3x+1)^6(1-4x)$

(C) $2(3x+1)^6(1-5x)$

(D) $2(3x+1)^6(1-11x)$

(E) $2(3x+1)(1-4x)$

$$2(3x+1)^7 - 16x(3x+1)^6$$
$$= (3x+1)^6\left[2(3x+1) - 16x\right]$$
$$= 2(3x+1)^6\left[(3x+1) - 8x\right]$$
$$= 2(3x+1)^6\left[1 + 3x - 8x\right]$$
$$= 2(3x+1)^6(1-5x)$$

23. Let $f(x)=\frac{2}{x}$ and $g(x)=\frac{17}{x^2}+1$. Find and simplify $f(g(x))$ completely.

(A) $\frac{2}{17+x^2}$ (B) $\frac{x^2}{9}$ (C) $\frac{2x^2}{17+x^2}$

(D) $\frac{17x^2+4}{4}$ (E) $17x^2+1$

$$f(g(x)) = \frac{2}{\left(\frac{17}{x^2}+1\right)}$$

$$= \frac{2}{\frac{17}{x^2}+\frac{x^2}{x^2}}$$

$$= \frac{2}{\frac{17+x^2}{x^2}}$$

$$= \frac{2}{\frac{17+x^2}{x^2}}$$

$$= 2\cdot\frac{x^2}{17+x^2} = \frac{2x^2}{17+x^2}$$

24. Let $f(t)=\sqrt{3t-1}$. Find $f^{-1}(4)$.

(A) $\frac{17}{3}$ (B) $\sqrt{13}$ (C) 5 (D) $\sqrt{11}$ (E) 4

$$f(t)=\sqrt{3t-1}$$

$$x=\sqrt{3y-1}$$

$$x^2 = 3y-1$$

$$x^2+1 = 3y$$

$$\frac{x^2+1}{3} = y$$

$$\therefore f^{-1}(t) = \frac{t^2+1}{3}$$

$$f^{-1}(4) = \frac{(4)^2+1}{3}$$

$$= \frac{16+1}{3}$$

$$= \frac{17}{3}$$

25. The function $S = f(b)$ gives a student's score on a standardized test as a function of the number of books b the student has read. If S is invertible, what does $f^{-1}(35) = 20$ mean?

(A) The student's score increases by 35/20 for every additional book the student reads.

(B) When the student's score is 20, the student has read 35 books.

(C) The student's score increases by 20/35 for every additional book the student reads.

(D) When the student's score is 35, the student has read 20 books.

(E) There is not enough information to determine the meaning.

S = student's score
b = number of books read

$\left.\right\rangle S = f(b)$

$f^{-1}(S) = b$

$f^{-1}(35) = 20$

Student's score is the input
number of books read is the output

Use the following tables of values to answer questions 26, 27, 28, and 29. Assume the functions are continuous, have domain all real numbers, and the characteristics of the functions are represented in the table.

x	$f(x)$
-5	3
-1	3
0	3
1	3
5	3

x	$g(x)$
-4	6
-2	10
0	0
2	-10
4	-6

x	$h(x)$
-3	-7
-1	-5
0	2
1	5
3	7

x	$k(x)$
-8	-5
-4	-1
0	0
4	-1
8	-5

26. Which of the functions could be an odd function? odd functions: $-f(x) = f(-x)$

(A) $f(x)$ (B) $g(x)$ (C) $h(x)$ (D) $k(x)$ (E) None of these

$-f(5) \neq f(-5)$
$-3 \neq 3$

$-g(4) = g(-4)$
$-(-6) = 6$
$6 = 6$
$-g(2) = g(-2)$
$-10 = -10$

$-g(0) = g(-0)$
$-0 = 0$
$0 = 0$

$-h(3) = h(-3)$
$-(7) = -7$
$-h(0) \neq h(-0)$
$-(2) \neq 2$

$-k(-8) \neq k(8)$
$-(-5) \neq -5$
$5 \neq -5$

27. Find $h(k(-4))$.

(A) 5 (B) 0 (C) -7 (D) -1 (E) -5

$k(-4) = -1$
$h(-1) = -5$

28. Find the average rate of change of $g(x)$ over the interval $[-2, 2]$.

(A) 5 (B) 0 (C) -5 (D) -20 (E) undefined

$g(-2) = 10$
$g(0) = 0$
$g(2) = -10$

rate of change $= \dfrac{\text{change in } y}{\text{change in } x}$

$R = \dfrac{\Delta y}{\Delta x} = \dfrac{-10-10}{2-(-2)} = \dfrac{-20}{4} = -5$

29. Find $(k-g)(4)$.

(A) 5 (B) -7 (C) 20 (D) -28 (E) -5

$(k-g)(4) = k(4) - g(4)$
$= (-1) - (-6)$
$= -1 + 6$
$= 5$

30. Which statement is true about the function $f(x) = -3(x-p)^2 + q$, provided that $p \neq q$?

(A) q is the maximum value of $f(x)$
(B) p is the maximum value of $f(x)$
(C) q is the minimum value of $f(x)$
(D) p is the minimum value of $f(x)$

general form for a quadratic function: $f(x) = a(x-h)^2 + k$

(h,k) is the vertex

'a' tells us if it opens up or down

+a opens up ∪

−a opens down ∩

a in our example is negative so it opens down with a vertex at point (p, q)

when $x = p$, the function is at its maximum, q.

31. Determine whether $f(x) = 2x^2 - 6x + k$ has a maximum or minimum value.

(A) Maximum value: k

(B) Minimum value: k

(C) Maximum value: $k - \frac{9}{2}$

(D) Minimum value: $k - \frac{9}{2}$

(E) Minimum value: $k + \frac{9}{2}$

$f(x) = ax^2 + bx + c$; $a = 2$, $b = -6$, $c = k$

a is positive so the parabola opens up ∪ which means the vertex is a minimum.

The x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-(-6)}{2(2)} = \frac{6}{4} = \frac{3}{2}$

$f(\frac{3}{2}) = 2(\frac{3}{2})^2 - 6(\frac{3}{2}) + k = 2(\frac{9}{4}) - \frac{18}{2} + k = \frac{9}{2} - \frac{18}{2} + k = -\frac{9}{2} + k = k - \frac{9}{2}$

32. Solve the equation $6x(x-1) + 5x = 2$.

(A) $x = \frac{-2}{3}$, $x = \frac{1}{2}$

(B) $x = 1$, $x = \frac{1}{2}$

(C) $x = \frac{2}{3}$, $x = \frac{-1}{2}$

(D) $x = 0$, $x = \frac{-1}{2}$

(E) $x = \frac{1}{3}$, $x = 3$

$6x(x-1) + 5x = 2$

$6x^2 - 6x + 5x = 2$

$6x^2 - x - 2 = 0$

$(3x - 2)(2x + 1) = 0$

$3x - 2 = 0$ $2x + 1 = 0$

$3x = 2$ $2x = -1$

$x = \frac{2}{3}$ $x = -\frac{1}{2}$

$x = \frac{2}{3}, -\frac{1}{2}$

33. Suppose the graph of a polynomial function $y = f(x)$ has the following end behavior:

$$y \to -\infty \quad \text{as} \quad x \to \infty$$
$$y \to -\infty \quad \text{as} \quad x \to -\infty$$

Which ONE of the following statements must be true?

(A) The degree of $f(x)$ is an odd number.
(B) $f(x)$ is an even function.
(C) $f(x)$ has a minimum value.
(D) The range of $f(x)$ is all real numbers.
(E) The leading coefficient of $f(x)$ is a negative number.

Since $y \to \infty$ as $x \to \infty$ and $x \to -\infty$, the function has an even degree.

Since $y \to -\infty$ as $x \to \infty$, the leading coefficient must be negative.

34. Solve for x: $\frac{x^2-6x}{x+2} = 0$

(A) $x = 0, x = 6,$ or $x = -2$ only
(B) $x = 6$ only
(C) $x = 0$ or $x = 6$ only
(D) $x = -2$ only
(E) $x = 6$ or $x = -2$ only

x+2 cannot equal 0, as that would make the fraction undefined.

$x^2 - 6x = 0$

$x(x-6) = 0$

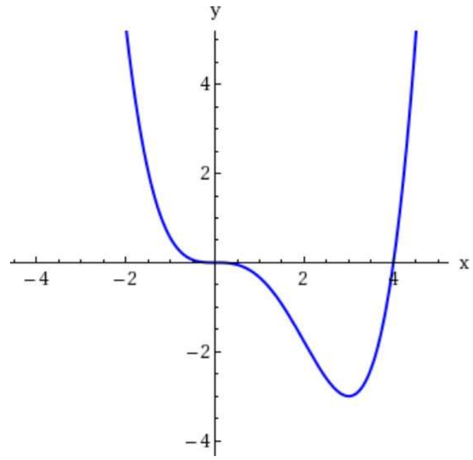
$x = 0 \qquad x - 6 = 0$

$\qquad\qquad x = 6$

$x = 0$ or $x = 6$ only

35. The graph of a polynomial function is given at the right. Assume all the characteristics of the polynomial are shown.

Which of the following could be a possible equation for the polynomial with $k > 0$?



(A) $p(x) = -kx^3(x-4)$

(B) $p(x) = kx^2(x-4)^2$

(C) $p(x) = -kx(x-4)$

(D) $p(x) = kx(x-4)$

(E) $p(x) = kx^3(x-4)$

Because the graph takes on shape, it is not linear or quadratic so we can eliminate options C & D. Because it opens up, the coefficient with k must be positive, so we can eliminate option A. For option B, since both of the x terms are being squared, $p(x)$ cannot be < 0 and the graph has negative $p(x)$ values, so the correct choice is E.

36. Find the value of A so that $y = -4$ is the horizontal asymptote of $g(x) = \dfrac{3x+7}{Ax-2}$.

(A) $A = -\dfrac{4}{3}$

(B) $A = -\dfrac{3}{4}$

(C) $A = -\dfrac{1}{2}$

(D) $A = -2$

(E) None of these

Simplify the leading coefficients to find the horizontal asymptote.

$$g(x) = \frac{3x+7}{Ax-2} \rightarrow \frac{3x}{Ax} = \frac{3}{A}$$ is the horizontal asymptote

$$\frac{3}{A} = -4$$

$$3 = -4A$$

$$-\frac{3}{4} = A$$

37. Find the value of p so that the vertical asymptote of $f(x)=\dfrac{6px}{4x+p}$ is $x=5$.

(A) $p=\dfrac{10}{3}$ (B) $p=-20$ (C) $p=-\dfrac{5}{4}$

(D) $p=10$ (E) None of these

The vertical asymptote is where the function is undefined, where the denominator is equal to 0.

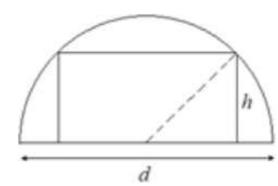
$4x+p=0$

$4(5)+p=0$

$20+p=0$

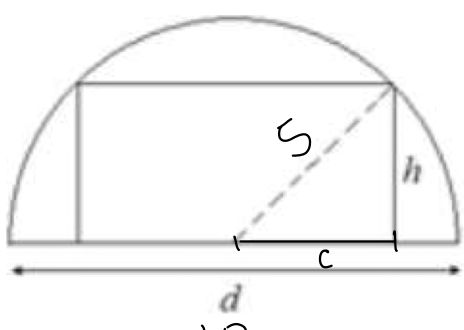
$p=-20$

38. A rectangle is inscribed in a semicircle with diameter 10 centimeters as shown. Express the area of the rectangle as a function of the height of the rectangle.



(A) $A(h)=2h\sqrt{25-h^2}$ (B) $A(h)=h\sqrt{10-h^2}$

(C) $A(h)=2h\sqrt{5-h^2}$ (D) $A(h)=2h\sqrt{10-h^2}$ (E) $A(h)=h\sqrt{25-h^2}$



If the diameter is 10, the radius is 5. The dotted line shows the radius. Using pythagorean theorem, we can find half of the length of the rectangle, c.

$h^2+c^2=5^2$

$c^2=5^2-h^2$

$c=\sqrt{25-h^2}$

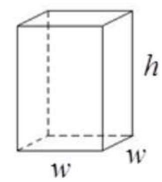
The entire length of of rectangle is 2c. $2\sqrt{25-h^2}$. The height is equal to the width.

Area = length · width

$A(h)=2\sqrt{25-h^2}\cdot h$

$A(h)=2h\sqrt{25-h^2}$

39. An open top rectangular box with a square bottom has a volume of 120 cubic meters. Its bottom and sides are made from two different materials. It costs 10 dollars per square meter for the bottom material and 12 dollars per square meter for the sides. Determine a model for cost of materials as a function of w.



(A) $C(w) = 20w^2 + \dfrac{5760}{w}$ (B) $C(w) = 10w^2 + \dfrac{11520}{w}$

(C) $C(w) = 10w^2 + \dfrac{5760}{w}$ (D) $C(w) = 20w^2 + \dfrac{1440}{w}$ (E) $C(w) = 10w^2 + \dfrac{1440}{w}$

Volume = length · width · height

$V = w^2 h = 120 \text{ m}^3 \rightarrow h = \frac{120}{w^2}$ m

Cost of the bottom = $10 · $w^2 = 10w^2$

cost of one side $\$12 \cdot h \cdot w = 12\left(\frac{120}{w^2}\right)(w)$

$= 12\left(\frac{120}{w}\right)$

$= \frac{1440}{w}$

cost of 4 sides = $4 \cdot \left(\frac{1440}{w}\right)$

$= \frac{5760}{w}$

cost of materials = cost of bottom + cost of 4 sides

$C(w) = 10w^2 + \frac{5760}{w}$

40. The police can determine the speed, *S,* that a car was traveling from the length of the skid mark, *L*, that the car leaves. Assuming *S* varies directly with the square root of *L,* express *S* as a function of *L.*

S(L) k $\sqrt{L}$

(A) $S(L) = kL^2$ (B) $S(L) = k\sqrt{L}$ (C) $L(S) = kS^2$

(D) $S(L) = k\sqrt{S}$ (E) $L(S) = kS$

41. The distance traveled by a falling object is directly proportional to the square of the time it takes to fall that far. If the object falls 100 feet in 2.5 seconds, how far does it fall in 5 seconds?

(A) 800 feet (B) 400 feet (C) 325 feet

(D) 250 feet (E) 200 feet

d in feet
t in seconds

$d = kt^2$
$100 = k(2.5^2)$
$100 = k(6.25)$
$16 = k$

$d = kt^2$
$d = 16(5^2)$
$d = 16(25)$
$d = 400$

Use the story below to answer questions 42 and 43.

A printer was purchased for P_0 dollars in 2016. The value of the printer in dollars, *P*, can be expressed as a function of *t*, the number of years since 2016.

42. Write a formula for *P* if the value of the printer decreases by $15 every year.

(A) $P(t) = P_0 - 15t$ (B) $P(t) = 15t - P_0$ (C) $P(t) = P_0(1-15t)$

(D) $P(t) = P_0(t-15)$ (E) $P(t) = P_0 + (t-15)$

P(t) is the value of the printer after t years.
The value of the printer starts at the purchase price, P_0, at the time of purchase. After 1 year, the value decreases \$15, so $P(1) = P_0 - 15$. After 2 years, the value decrease \$15 for the first year and \$15 for the second year, so $P(2) = P_0 - 15(2)$.
$P(t) = P_0 - 15t$

43. Write a formula for *P* if the value of the printer decreases by 7% each year.

(A) $P(t) = P_0(0.07)^t$ (B) $P(t) = P_0 - (0.07)^t$ (C) $P(t) = P_0(0.93)^t$

(D) $P(t) = P_0(0.3)^t$ (E) $P(t) = P_0 - P_0(0.93)^t$

P(t) is the value of the printer after t years.
The value of the printer starts at the purchase price, P_0, at the time of purchase. After 1 year, the value is 7% less, which means the printer retains 93% of it's value from the previous year. After 1 year, the value of the printer is $P(1) = P_0(0.93)$. After 2 years, the printer is valued at 93% of its year 1 value, so $P(2) = (P_0(0.93))(0.93) = P_0(0.93)^2$. We can repeat this pattern and find that $P(t) = P_0(0.93)^t$.

44. Solve for y: $xy + a = x^3 + \frac{1}{a} y$

(A) $y = \dfrac{ax^3 - a^2}{ax - 1}$ (B) $y = \dfrac{x^3 - a^2}{x - 1}$ (C) $y = \dfrac{ax^3 - a^2}{x - 1}$

(D) $y = \dfrac{ax - 1}{ax^3 - a^2}$ (E) $y = \dfrac{ax^3 - a}{ax - 1}$

$xy + a = x^3 + \frac{1}{a} y$ multiply everything by 'a' to get rid of the fraction

$xya + a^2 = x^3 a + y$ get 'y' terms on one side, and everything else on the other side

$xya - y = x^3 a - a^2$ factor out the 'y'

$y(xa - 1) = x^3 a - a^2$ divide both sides by $(xa - 1)$ to isolate the 'y'

$$y = \frac{x^3 a - a^2}{xa - 1} = \frac{ax^3 - a^2}{ax - 1}$$

45. Solve the inequality $\dfrac{(9y + 11)(y - 6)}{y^2} \le 0$.

(A) $\left(-\infty, -\frac{11}{9}\right] \cup [6, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$ (C) $(0,6]$

(D) $[-\frac{11}{9}, 0) \cup (0,6]$ (E) $[-\frac{11}{9}, 6]$

$y^2 \neq 0$

$y \neq 0$

$9y + 11 = 0$

$9y = -11$

$y = -\frac{11}{9}$

$y - 6 = 0$

$y = 6$

create test intervals to see which ones make the inequality true.

$y \le -\frac{11}{9}$ False

$-\frac{11}{9} \le y < 0$ True

$0 < y \le 4$ True

$y \ge 6$ False

$-\frac{11}{9} \le y < 0$ or $0 < y \le 6$

$[-\frac{11}{9}, 0) \cup (0, 6]$

46. Use the information below to find the vertical asymptote(s) of $f(x) = \dfrac{n(x)}{d(x)}$

$n(x)$ is a quadratic function with zeroes $x = 6$ and $x = 18$

$d(x)$ is a linear function with zero $x = 9$

(A) None (B) $x = 0$ (C) $x = 6, x = 18, x = 9$

(D) $x = 6, x = 18$ (E) $x = 9$

The vertical asymptote is where the function is undefined, where the denominator is equal to 0.
d(x) is undefined at x = 9, so there is a vertical asymptote at x = 9.

47. Suppose $\cot\theta > 0$ and $\sec\theta < 0$. In which quadrant could θ terminate?

(A) Quadrant I (B) Quadrant II

(C) Quadrant III (D) Quadrant IV

$\cot\theta > 0$

$\dfrac{\cos\theta}{\sin\theta} > 0$

$\sec\theta < 0$

$\dfrac{1}{\cos\theta} < 0$

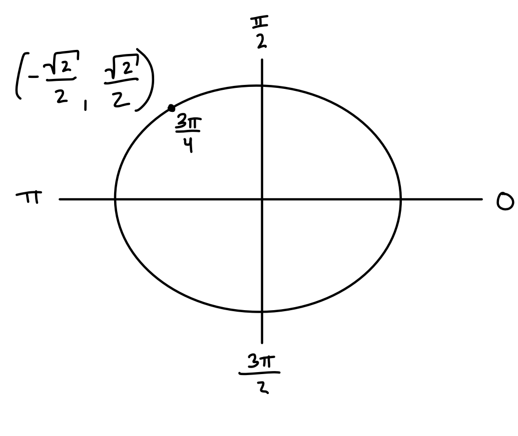
$\cos\theta$ is negative which means θ must be in the second or third quadrant.

Since $\cos\theta < 0$, $\sin\theta$ must be less than 0 for $\cot\theta > 0$.
if $\sin\theta$ and $\cos\theta$ are both negative, θ is in Quadrant III.

48. Find the terminal point (x,y) on the unit circle determined by the real number $t = \dfrac{3\pi}{4}$

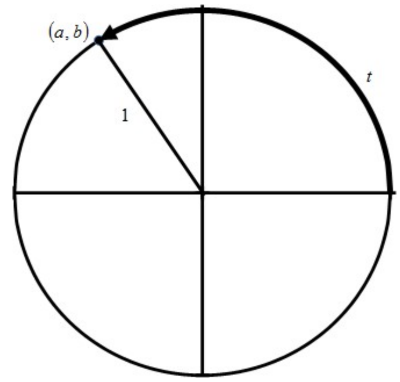
(A) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ (B) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ (C) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

(D) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ (E) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



Use the information and graph to answer questions 49 and 50.

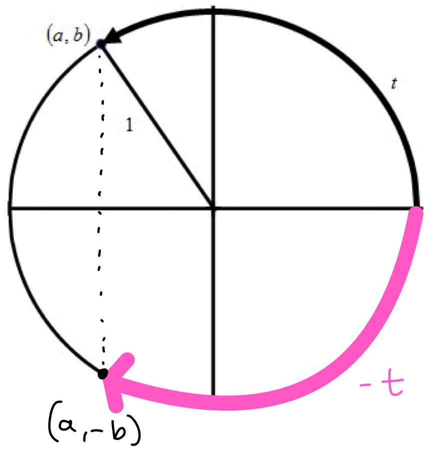
Suppose a real number t determines the terminal point (a, b) on the unit circle. See the graph at the right.



49. Find the terminal point determined by the real number $-t$.

(A) (b, a) (B) $(-a, b)$ (C) (a, b)

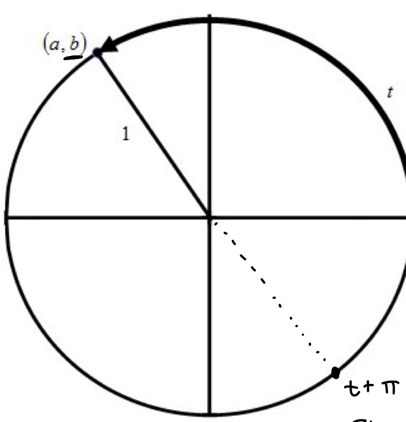
(D) $(-a, -b)$ (E) $(a, -b)$



50. Find $\sin(t+\pi)$.

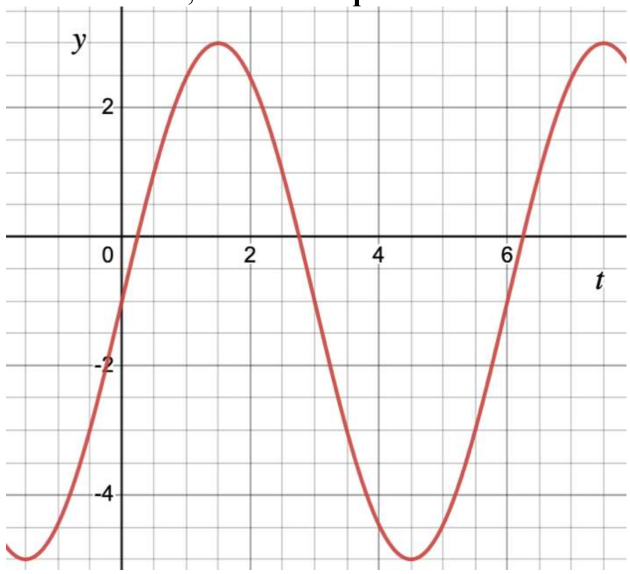
(A) b (B) a (C) $-b$ (D) $-a$ (E) None of these

$\sin(t) = b$



$\sin(t+\pi) = -b$

Use the graph of $f(t) = A\sin(Bt) + C$, to answer questions 51 and 52.



51. Determine the value of B.

(A) $B = \dfrac{\pi}{3}$ (B) $B = \dfrac{\pi}{2}$ (C) $B = 6$ (D) $B = 3\pi$ (E) $B = 12$

The period is the length of one cycle which is 6 units in this example. $6 = \frac{2\pi}{B}$

$6B = 2\pi$

$B = \frac{2\pi}{6}$

$B = \frac{\pi}{3}$

52. Determine the value of C.

(A) $C = 6$ (B) $C = 0$ (C) $C = -1$ (D) $C = 4$ (E) $C = -5$

C is the vertical shift of the equilibrium line from the horizontal axis. The equilibrium line is at $y = -1$, so $C = -1$.

53. In a region of Australia, the population of a particular type of kangaroo is modeled by the function $P(t) = 1300 - 140\sin(2t)$, where t is measured in years. According to the model, what is the maximum kangaroo population?

(A) 1160 (B) 1440 (C) 1300 (D) 1580 (E) There is no largest number.

sin(2t) can take on values between -1 and 1.

$1300 - 140(1) = 1300 - 140 = 1160$
$1300 - 140(0) = 1300 - 0 = 1300$
$1300 - 140(-1) = 1300 + 140 = 1440$

1440 is the highest value

54. The minimum value of $g(x) = -37\cos(x-3) + 21$ is

(A) -58 (B) -16 (C) 37 (D) 21 (E) 16

cos(x−3) can take on values between -1 and 1.

$-37(-1) + 21 = 37 + 21 = 58$
$-37(0) + 21 = 0 + 21 = 21$
$-37(1) + 21 = -37 + 21 = -16$

-16 is the lowest value

55. Which ONE of the following is a vertical asymptote of the graph of $f(x) = \tan(x + \frac{\pi}{3})$?

(A) $x = -\dfrac{\pi}{3}$ (B) $x = -\dfrac{\pi}{6}$ (C) $x = 0$ (D) $x = \dfrac{\pi}{6}$ (E) $x = \dfrac{2\pi}{3}$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ The vertical asymptote is where the function is undefined, where the denominator is equal to 0. $\cos(x) = 0$ when $x = \frac{\pi}{2} + n\pi$, where n is an integer.

$x + \frac{\pi}{3} = \frac{\pi}{2}$
$x = \frac{3\pi}{6} - \frac{2\pi}{6}$
$x = \frac{\pi}{6}$

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56. The domain of $f(t)=\cos^{-1}(t)$ is:

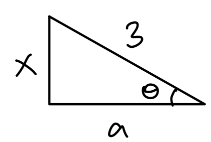
(A) $[0,2\pi]$ (B) $[-\frac{\pi}{2},\frac{\pi}{2}]$ (C) $(-\infty,\infty)$ (D) $[0,\pi]$ (E) $[-1,1]$

cosine only takes on values between -1 & 1.
The domain of $f(t)=\cos^{-1}(t)$ is $[-1,1]$.

57. Simplify the expression $\tan(\sin^{-1}(\frac{x}{3}))$. Assume $0<x<3$.

(A) $\frac{x}{3-x}$ (B) $\frac{x}{3}$ (C) $\frac{\sqrt{x^2-9}}{3}$ (D) $\frac{x}{\sqrt{9-x^2}}$ (E) $\frac{\sqrt{9-x^2}}{x}$

$\sin^{-1}(\frac{x}{3})=\theta$ $\therefore$ $\frac{x}{3}=\sin\theta$



$3^2=x^2+a^2$
$9=x^2+a^2$
$9-x^2=a^2$
$\sqrt{9-x^2}=a$

$\tan(\sin^{-1}(\frac{x}{3}))=\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}=\frac{x}{\sqrt{9-x^2}}$

58. Find the length of an arc that subtends a central angle of $135°$ in a circle with radius 5. The length of the arc is:

(A) 675π (B) 675 (C) $\frac{27\pi}{4}$ (D) $\frac{3\pi}{20}$ (E) $\frac{15\pi}{4}$

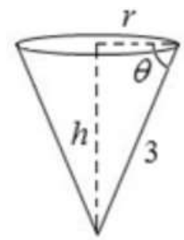
$L=r*\theta$

arc length = radius * central angle (in radians)

$135\times\frac{\pi}{180}=\frac{3\pi}{4}$

$L=5(\frac{3\pi}{4})=\frac{15\pi}{4}$

59. Express the volume of a cone as a function of θ.
(The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)



(A) $V = 9\pi \sin^2\theta \cos\theta$
(B) $V = 9\pi \cos^2\theta \sin\theta$
(C) $V = 3\pi \cos^2\theta \sin\theta$
(D) $V = 3\pi \sin\theta \cos\theta$
(E) $V = 3\pi \sin^2\theta \cos\theta$

$V = \frac{1}{3}\pi r^2 h$

r is adjacent, h is opposite, and hypotenuse is 3.

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{r}{3}$ $\quad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{3}$

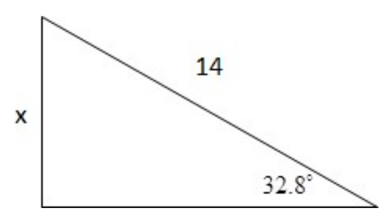
$3\cos\theta = r$ $\quad 3\sin\theta = h$

$V = \frac{1}{3}\pi (3\cos\theta)^2 (3\sin\theta)$

$V = \frac{1}{\cancel{3}}\pi\, 9\cos^2\theta\, \cancel{3}\sin\theta$

$V = 9\pi\cos^2\theta\sin\theta$

60. Use the angle $32.8°$ to determine the exact value of x in the figure below.



(A) $x = 14\sin(32.8°)$
(B) $x = \frac{\cos(32.8°)}{14}$
(C) $x = 14\tan(32.8°)$
(D) $x = \frac{14}{\sin(32.8°)}$
(E) $x = 14\cos(32.8°)$

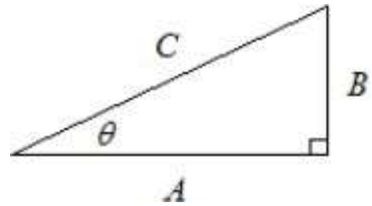
$\theta = 32.8°$, x is opposite, and hypotenuse is 14

$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin(32.8°) = \frac{x}{14}$

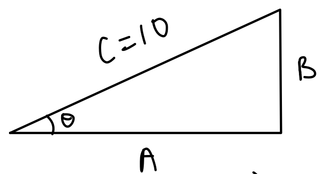
$14\sin(32.8°) = x$

61. Consider a right triangle with an acute angle given by $\theta = arcsin(\frac{2}{5})$, as shown below.



If the hypotenuse of the triangle has length 10, determine the length of side A.

(A) $2\sqrt{21}$ (B) $\sqrt{21}$ (C) $\sqrt{29}$ (D) $2\sqrt{26}$ (E) 2



$\theta = arcsin\left(\frac{2}{5}\right)$

$\sin(\theta) = \frac{2}{5}$

$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

$\frac{2}{5} = \frac{B}{10}$

$4 = B$

$A^2 + B^2 = C^2$

$A^2 + 4^2 = 10^2$

$A^2 + 16 = 100$

$A^2 = 84$

$A = \sqrt{84}$

$A = \sqrt{4 \cdot 21}$

$A = 2\sqrt{21}$

62. Let $\cos\phi = -0.4$. Determine the value of $\cos(-\phi)$.

(A) -0.4
(B) $\pi - 0.4$
(C) $\pi + 0.4$
(D) $2\pi - 0.4$
(E) 0.4

$\cos(\theta) = \cos(-\theta)$

$\cos(\theta) = -0.4 \quad \therefore \cos(-\theta) = -0.4$

63. Which ONE of the following angles is coterminal with -245?

(A) $-115°$ (B) $25°$ (C) $65°$ (D) $115°$ (E) $245°$

$360° - 245° = 115°$

64. What can we say about the y-values of the graph of $f(x) = 13x(x+2)^3$ as $x \to -\infty$?

(A) $y \to -\infty$ (B) $y \to \infty$ (C) $y \to 2$

(D) $y \to 13$ (E) $y \to 0$

As $x \to -\infty$, $(x+2)^3 \to -\infty$

As $x \to -\infty$, $13x \to -\infty$

A really large negative number multiplied by a really large negative number gives us a really large positive number. $y \to \infty$

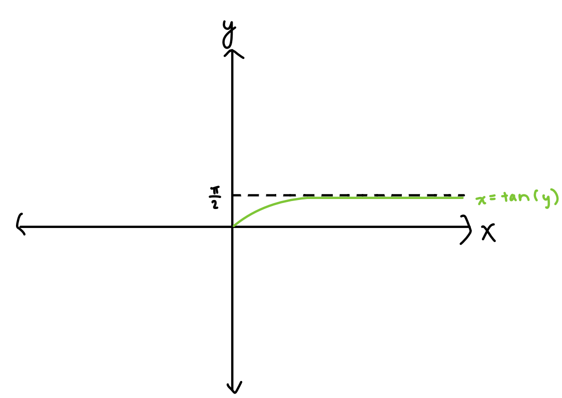
65. What can we say about the y-values of $tan^{-1}(x)$ as $x \to \infty$?

(A) $y \to \infty$ (B) $y \to \frac{\pi}{4}$ (C) $y \to \frac{\pi}{2}$

(D) $y \to \frac{-\pi}{2}$ (E) $y \to 0$

$y = \tan^{-1}(x)$

$\tan(y) = x$



66. Simplify the expression $\sin A(\csc A - \sin A)$.

(A) $1-\cos A$ (B) $\sin^2 A$ (C) 1

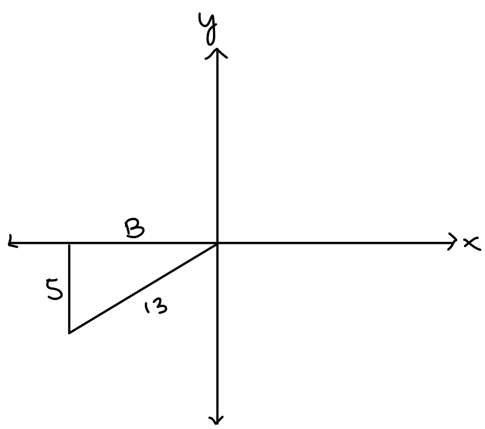
(D) $1-\sin A$ (E) $\cos^2 A$

$$\sin A(\csc A - \sin A)$$
$$= \sin A\left(\frac{1}{\sin A} - \sin A\right)$$
$$= \frac{\sin A}{\sin A} - \sin^2 A$$
$$= 1 - \sin^2 A \quad = \cos^2 A$$

67. Suppose $\sin(x) = -5/13$, where x terminates in Quadrant III. Find $\sin(2x)$.

(A) $\frac{120}{169}$ (B) $\frac{8}{13}$ (C) $-\frac{10}{13}$

(D) $-\frac{120}{169}$ (E) $-\frac{8}{13}$



Trig identity : $\sin 2x = 2\sin x \cos x$

$\sin(x) = -\frac{5}{13}$
$\cos(x) = -\frac{12}{13}$ since we are in Quadrant 3

$$\sin(2x) = 2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)$$
$$= 2\left(\frac{60}{169}\right)$$
$$= \frac{120}{169}$$

pythagorean theorem

$5^2 + B^2 = 13^2$

$25 + B^2 = 169$

$B^2 = 144$

$B = 12$

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68. Solve for t: $(2\cos t - 1)(\cos t - 2) = 0$ on the interval $0 \le t < 2\pi$.

(A) $t = 0,\ t = \pi,\ t = \frac{\pi}{3}$, or $t = \frac{5\pi}{3}$ only

(B) $t = 0$ or $t = \pi$ only.

(C) $t = \frac{\pi}{3}$, or $t = \frac{5\pi}{3}$ only

(D) $t = \frac{\pi}{6}$, or $t = \frac{11\pi}{6}$ only

(E) $t = 0,\ t = \pi,\ t = \frac{\pi}{6}$, or $t = \frac{11\pi}{6}$ only

$(2\cos t - 1)(\cos t - 2) = 0 \qquad 0 \le t < 2\pi$

$2\cos t - 1 = 0$
$2\cos t = 1$
$\cos t = \frac{1}{2}$
$t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\cos t - 2 = 0$
$\cos t = 2$
No solution

69. Solve for x: $\frac{\sin(3x) - 2}{\cos 3x} = 0$ on the interval $0 \le x < \frac{\pi}{2}$

(A) $x = \frac{\pi}{6}$ only

(B) $x = \frac{\tan^{-1}(2)}{3}$ only

(C) $x = 0$ only

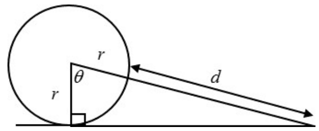
(D) $x = \sin^{-1}(\frac{2}{3})$ only

(E) No solution

$\frac{\sin(3x) - 2}{\cos 3x} = 0 \qquad 0 \le x < \frac{\pi}{2}$

$\sin(3x) - 2 = 0$
$\sin(3x) = 2 \qquad -1 \le \sin(3x) \le 1$
No solution

70. The radius of the circle below is 18 inches. Express the length d as shown in terms of θ.



(A) $d(\theta) = \dfrac{18}{\cos(\theta)} - 18$

(B) $d(\theta) = \dfrac{18}{\cos(\theta)} + 18$

(C) $d(\theta) = 18\cos(\theta) - 18$

(D) $d(\theta) = 18\cos(\theta) + 18$

(E) $d(\theta) = \dfrac{18}{\cos(\theta)}$

$h = d + r$

$r = 18$ inches

$\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{r}{h} = \dfrac{18}{h}$

$\cos(\theta) = \dfrac{18}{h}$

$h\cos(\theta) = 18$

$h = \dfrac{18}{\cos\theta}$

$d + 18 = \dfrac{18}{\cos\theta}$

$d(\theta) = \dfrac{18}{\cos\theta} - 18$

71. The range of $f(t) = -3^t + 200$ is:

(A) $(-\infty, \infty)$ (B) $(-\infty, 0)$ (C) $(-\infty, 200)$

(D) $(200, \infty)$ (E) $(0, \infty)$

$f(t) = -3^t + 200$

$y = f(t)$

$y = -3^t + 200$

$y + 3^t = 200$

$3^t = 200 - y$

$t = \log_3(200 - y)$

$\log_3(200 - y) > 0$

$200 - y > 0$

$200 > y$

$(-\infty, 200)$

72. Find the inverse function of $h(t) = 7^t + 19$.

(A) $h^{-1}(t) = \log_{19}(t - 7)$ (B) $h^{-1}(t) = \log_7(t - 19)$

(C) $h^{-1}(t) = -19 + 7^t$ (D) $h^{-1}(t) = -19 + \log_7(t)$

(E) $h^{-1}(t) = \dfrac{1}{7^t + 19}$

To find the inverse, solve for t.

$h(t) = 7^t + 19$

$h = 7^t + 19$

$h - 19 = 7^t$

$\log_7(h - 19) = t$

$h^{-1}(t) = \log_7(t - 19)$

Use the following story to answer questions 73 and 74.

The velocity of a skydiver, in feet per second, t seconds after jumping out of an airplane, is modeled by the function $v(t)=a(1-e^{-bt})$, where a and b are positive constants.

73. Based on this model, what happens to the skydiver's velocity as $t \to \infty$? The skydiver's velocity approaches:

(A) ∞ (B) $a+b$ (C) $a-b$ (D) a (E) b

$v(t) = a(1-e^{-bt})$

as $t \to \infty$, $-e^{-bt} \to 0$, so we are just left with a.

74. Assume that $a=100$. If the skydiver's velocity is 70 feet per second after 10 seconds, determine the exact value of b.

(A) $b=\dfrac{\ln(10)}{70}$ (B) $b=\dfrac{\ln(0.7)}{10}$ (C) $b=\dfrac{\ln(0.7)}{-10}$

(D) $b=\dfrac{\ln(0.3)}{10}$ (E) $b=\dfrac{\ln(0.3)}{-10}$

$v(t) = a(1-e^{-bt})$ $a=100$ $v=70$ $t=10$

$70 = 100(1-e^{-b(10)})$

$70 = 100(1-e^{-10b})$

$\frac{7}{10} = 1 - e^{-10b}$

$0.7 + e^{-10b} = 1$

$e^{-10b} = 0.3$

$\ln(e^{-10b}) = \ln(0.3)$

$-10b = \ln(0.3)$

$b = \dfrac{\ln(0.3)}{-10}$

75. Simplify the expression completely: $\ln(4e^x)$

(A) $4x$ (B) $\ln(4) + x$ (C) $\ln(x) + 4x$

(D) $x\ln(4) + x$ (E) $\ln(4) + e^x$

$\ln(4e^x) = \ln(4 \cdot e^x) = \ln(4) + \ln(e^x) = \ln(4) + x$

Use $f(x) = \log_7(11x+3)$ **to answer questions 76 and 77.**

76. Find the domain of $f(x)$.

(A) $[-\frac{3}{11}, \infty)$ (B) $(-\frac{11}{3}, \infty)$ (C) $(0, \infty)$

(D) $(-\frac{3}{11}, \infty)$ (E) $(\frac{3}{11}, \infty)$

$\log_7(11x+3) > 0$

$11x + 3 > 0$

$11x > -3$

$x > -\frac{3}{11}$

77. Find the exact zero of $f(x)$.

(A) $x = -\frac{2}{11}$ (B) $x = -\frac{4}{11}$ (C) $x = \frac{10}{11}$

(D) $x = \log_7(3)$ (E) None of these

$0 = \log_7(11x+3)$

$7^0 = 7^{\log_7(11x+3)}$

$1 = 11x + 3$

$-2 = 11x$

$-\frac{2}{11} = x$

78. Solve for x: $\log_4(2x+1) - \log_4(x-3) = 1$

(A) $x = -2$ only (B) $x = 4$ or $x = \frac{7}{2}$ only (C) $x = \frac{11}{2}$ only

(D) $x = -4$ only (E) $x = \frac{13}{2}$ only

$\log_4(2x+1) - \log_4(x-3) = 1$

$= \log_4\left(\frac{2x+1}{x-3}\right) = 1$

$= 4^{\log_4\left(\frac{2x+1}{x-3}\right)} = 4^1$

$= \frac{2x+1}{x-3} = 4 \rightarrow 2x+1 = 4(x-3)$

$2x+1 = 4x-12$

$2x+13 = 4x$

$13 = 2x$

$\frac{13}{2} = x$

79. Let $f(x) = \log_3(9x)$ and $g(x) = 3^x$. Find $f(g(x))$ and simplify.

(A) $2+x$ (B) $3+x$ (C) $9+x$

(D) $2x$ (E) $9x$

$f(x) = \log_3(9x)$

$g(x) = 3^x$

$f(g(x)) = \log_3(9 \cdot 3^x)$

$= \log_3(9) + \log_3(3^x)$

$= 2 + x$

$\log_3(9)$

$\log_a b = c$

$a^c = b$

$3^c = 9$

$c = 2$

$\log_3(9) = 2$

80. Solve for k: $11ke^{2k} + 9k^2e^{2k} = 0$.

(A) $k = 0$ only (B) $k = -\frac{11}{9}$ only (C) $k = 0, k = -\frac{11}{9}$ only

(D) $k = \ln(2)$ only (E) $k = 0, k = \ln(2), k = -\frac{11}{9}$ only

$11Ke^{2K} + 9K^2e^{2K} = 0$

$K(11e^{2K} + 9Ke^{2K}) = 0 \longrightarrow K = 0$

$Ke^{2K}(11 + 9K) = 0 \longrightarrow 11 + 9K = 0$

$9K = -11$

$K = \frac{-11}{9}$

$e^{2K} = 0$

$\ln(e^{2K}) = \ln(0)$

Not a solution

81. Let $f(x) = C \cdot b^x$. Determine the constants C and b so that $f(3) = 7$ and $f(4) = 35$.

(A) $C = \frac{125}{7}, b = 7$ (B) $C = \frac{7}{125}, b = 5$ (C) $C = \frac{7}{125}, b = 7$

(D) $C = 3, b = 35$ (D) $C = 7, b = 5$

$f(x) = C \cdot b^x$

$f(3) = 7$ and $f(4) = 35$

$7 = C \cdot b^3$ $35 = C \cdot b^4$ $7 = C \cdot 5^3$

$\frac{7}{b^3} = C$ $7 = C \cdot 125$

$35 = \frac{7}{b^3} \cdot b^4$ $\frac{7}{125} = C$

$35 = \frac{7b^4}{b^3}$

$35 = 7b$

$5 = b$

Use the story to answer questions 81 and 82.

A population grows with an annual growth rate of 16.6% per year.

82. What is the population's continuous growth rate per year? Round to one decimal place.

(A) 16.6% (B) 2.8% (C) 18.1% (D) 15.4% (E) 14.6%

$P = P_0 e^{rt}$

r = continuous growth rate
t = time (1 year)
e = Euler number
P_0 = starting population
P = annual growth rate

$1.166 = 1e^{r(1)}$

$\ln(1.166) = r$

$0.153579 = r$

$15.4\% = r$

83. What is the population's annual growth factor?

(A) 16.6 (B) 1.66 (C) 1.166 (D) 0.154 (E) 1.536

84. Rewrite the following expression: $\log(x+2) - 5\log(x^2+1) + 3\log(x)$

(A) $\log\left(\frac{x^3(x+2)}{(x^2+1)^5}\right)$ (B) $\frac{\log(x+2)\log(x^3)}{\log(x^2+1)^5}$ (C) $\log\left(\frac{x+2}{x^3(x^2+1)^5}\right)$

(D) $\frac{\log(x^3(x+2))}{\log(x^2+1)^5}$ (E) $-15\log\left(\frac{x(x+2)}{(x^2+1)}\right)$

$\log(x+2) - 5\log(x^2+1) + 3\log(x)$

$= \log(x+2) - \log(x^2+1)^5 + \log(x)^3$ power rule

$= \log(x^3(x+2)) - \log(x^2+1)^5$ product rule

$= \log\left(\frac{x^3(x+2)}{(x^2+1)^5}\right)$ quotient rule

85. Which of the following functions have at least one horizontal asymptote?

(1) $f(x) = \arctan x$ (2) $f(x) = 7\left(\frac{1}{5}\right)^x$ (3) $f(x) = 5x^5 + 7x^2 - 1$

(4) $f(x) = \sqrt{x+7}$ (5) $f(x) = \frac{x^3}{x^2+5}$

(A) (4) and (5) only (B) (1), (2), and (5) only

(C) (3) and (4) only (D) (1) and (2) only

(E) (2) and (5) only

(1) multiple horizontal asymptotes

(2) 1 horizontal asymptote at $y=0$

(3) no horizontal asymptotes

(4) no horizontal asymptotes

(5) no horizontal asymptotes

86. Each function below describes how something changes. Use the descriptions to determine which function(s) describe exponential growth or decay.

$f(t)$: The area of the circle doubles every 2 hours.

$g(t)$: The mass of the algae colony decreases by 2% each day.

$h(t)$: The volume of the sphere is proportional to its radius.

(A) $f(t)$ only (B) $g(t)$ only (C) $f(t)$ and $g(t)$ only

(D) $g(t)$ and $h(t)$ only (E) $f(t)$, $g(t)$ and $h(t)$

$y = a \cdot b^x$ represents exponential growth or decay

$f(t) = 2^x$

$g(t) = (0.02)^x$

$h(t) = kr$

87. A video is posted on the internet. By 1:00 pm today, there were 2500 views. By 5:00 pm today, there were 6500 views. Express the number of views, V, as a function of the number of hours since 1:00 pm today if the number of views increases exponentially.

(A) $V(t) = 6500(2.6)^{(t-1)/4}$ (B) $V(t) = 2500(2.6)^{t}$ (C) $V(t) = 2500(2.6)^{t/4}$

(D) $V(t) = 2500(2.6)^{(t-1)/4}$ (E) $V(t) = 6500(2.6)^{-t/4}$

1:00 pm 2500
5:00pm 6500

$f(x) = a \cdot b^x$ → $V(t) = 2500\,(2.6)^{t/4}$

a = initial amount = 2500

b = growth rate = $\frac{6500}{2500} = 2.6$

x = number of time intervals = $t/4$

1pm → 2pm, 2pm → 3pm, 3pm → 4pm, 4pm → 5pm

88. Suppose the number of "likes" for a particular Instagram page increases according to a model given by $V(t) = V_0 e^{0.08t}$, where V is measured in millions and t is measured in weeks.

How long will it take for the number of "likes" to triple?

(A) $\ln(37.5)$ weeks (B) $12.5\ln(3)$ weeks (C) $12.5\ln(3V_0)$ weeks

(D) $0.08\ln(3)$ weeks (E) $3\ln(1.08)$ weeks

$V(t) = V_0 e^{0.08t}$

$3V_0 = V_0 e^{0.08t}$

$3 = e^{0.08t}$

$\ln(3) = 0.08t$

$12.5\ln(3) = t$

89. Suppose $f(x) = \frac{x^2 - 4}{cx(2-x)}$. Determine the value of c so that $\lim_{x\to\infty} f(x) = 5$.

(A) c = 5 (B) c = –5 (C) c = 1/5

(D) c = –1/5 (E) c = 1

$$f(x) = \frac{x^2-4}{cx(2-x)} \qquad \lim_{x\to\infty} f(x) = 5$$

$$= \frac{x^2-4}{2cx - cx^2}$$

As $x\to\infty$, $f(x)$ behaves like $\frac{x^2}{-cx^2}$.

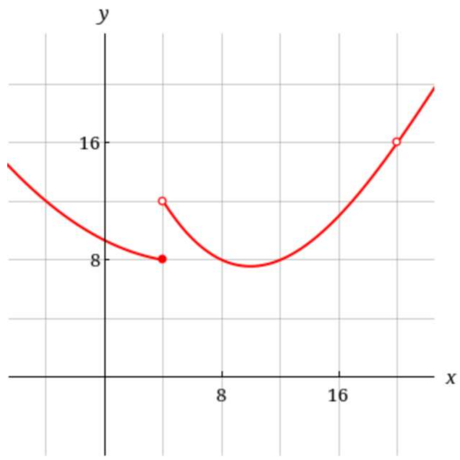
In order for the $\lim_{x\to\infty} f(x) = 5$, $\frac{1}{-c}$ must be 5.

$$5 = \frac{1}{-c}$$

$$-5c = 1$$

$$c = -\frac{1}{5}$$

Use the graph of *g(x)* **below to answer questions 90 and 91.**



90. Evaluate $\lim_{x \to 4^-} f(x)$.

(A) 4 (B) 8 (C) 12 (D) 16 (E) DNE

As x approaches 4 from the left, f(x) approaches 8.

91. Evaluate $\lim_{x \to 20} f(x)$.

(A) 8 (B) 12 (C) 16 (D) 20 (E) DNE

As x approaches 20 from both sides, f(x) approaches 16.

92. Suppose *g(t)* is an exponential function. All of the following statements **must** be true except for one. Which of the following statements could NOT be true?

(A) $g(t)$ has no vertical asymptote. True

(B) $g(t)$ has a horizontal asymptote. True

(C) $\lim_{t \to 0} g(t)$ exists (i.e. this limit is a real number) True

(D) $\lim_{t \to -\infty} g(t) = \lim_{t \to \infty} g(t)$ False

(E) $g^{-1}(t)$ is a logarithmic function. True